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Cellular automata models for synchronized traffic flow

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Abstract

This paper presents a new cellular automata model for describing synchronized traffic flow. The fundamental diagrams, the spacetime plots and the 1 min average data have been analysed in detail. It is shown that the model can describe the outflow from the jams, the light synchronized flow as well as heavy synchronized flow with average speed greater than approximately 24 km h^{-1}. As for the synchronized flow with speed lower than 24 km h^{-1}, it is unstable and will evolve into the coexistence of jams, free flow and light synchronized flow. This is consistent with the empirical findings (Kerner B S 1998 *Phys. Rev. Lett.* **81** 3797).

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1. Introduction

In the last few decades, traffic problems have attracted the interest of a community of physicists [1–3]. Traffic flow, a kind of many-body systems of strongly interacting vehicles, shows various complex behaviours. Numerous empirical data of the highway traffic have been obtained, which demonstrate the existence of qualitatively distinct dynamic states [4–7]. In particular, three distinct dynamic phases are observed on highways: the free traffic flow, the traffic jam and the synchronized traffic flow. It has been found out experimentally that the complexity in traffic flow is linked to diverse spacetime transitions between the three basically different kinds of traffic [4].

To understand the behaviour of traffic flow, various traffic flow models have been proposed and studied, including car-following models, cellular automaton (CA) models, gas-kinetic models and hydrodynamic models [8–16]. With the help of these models, free flow and jams are well understood. On the other hand, the nature of synchronized traffic flow remains unclear despite various efforts [4, 5, 16, 17]. Particularly, besides the 'three-phase theory' [18], there are other possible explanations such as heterogeneity of real traffic or simply a wrong interpretation of the measured traffic data [19].

Compared with other dynamical approaches, CA models are conceptually simpler, and can be easily implemented on computers for numerical investigations. So, they developed very quickly in the last decade after the first CA model was proposed in 1992 by Nagel and Schreckenberg (NS model) [9]. In the NS model, it is shown that start–stop waves appear in the congested traffic region as observed in real freeway traffic. However, the NS model cannot reproduce the synchronized flow. Later, several improved versions of the NS model were proposed, e.g., the slow-to-start models [10]. But they still cannot be used to simulate the synchronized flow.

Recently, Knospe *et al* proposed a more realistic CA model [20]. The model considers the desire of the drivers for smooth and comfortable driving (hereafter it is referred to as CD model). It was claimed that the CD model is able to reproduce the three phases and a good agreement with the empirical data can be found. Nevertheless, our simulations show that the CD model cannot reproduce the light synchronized flow, at least in a situation where the disorder effect and/or lane change behaviour are not considered (see section 2)[1].

Thus, in this paper, we try to deliver a better understanding of the synchronized flow by presenting a new CA model that can reproduce both the light and heavy synchronized flows even in a simple situation where the lane change behaviour and/or disorder effect are not considered. This model is based on the CD model, so we carry out the simulation of the CD model in section 2. In section 3, we present this new model and the numerical results are analysed. The conclusions are given in section 4.

2. The CD model

The CD model is defined as follows. First, the randomization function

$$p(v_n(t), b_{n+1}(t), t_h, t_s) = \begin{cases} p_b: & \text{if} \quad b_{n+1} = 1 \quad \text{and} \quad t_h < t_s \\ p_0: & \text{if} \quad v_n = 0 \\ p_d: & \text{in all other cases} \end{cases} \tag{1}$$

and the effective distance

$$d_n^{\text{eff}} = d_n + \max(v_{\text{anti}} - gap_{\text{safety}}, 0) \tag{2}$$

are introduced, where d_n is the gap of the vehicle n, v_n is the velocity of the vehicle n (here vehicle $n+1$ precedes vehicle n) and b_n is the status of the brake light (on (off)$\rightarrow$ $b_n = 1(0)$). The two times $t_h = d_n/v_n(t)$ and $t_s = \min(v_n(t), h)$, where h determines the range of interaction with the brake light, are introduced to compare the time t_h needed to reach the position of the leading vehicle with a velocity-dependent interaction horizon t_s. $v_{\text{anti}} = \min(d_{n+1}, v_{n+1})$ is the expected velocity of the preceding vehicle in the next time step and gap_{safety} controls the effectiveness of the anticipation. The parallel update rules consist of five steps.

1. Determination of the randomization parameter p:

$$p = p(v_n(t), b_{n+1}(t), t_h, t_s)$$
$$b_n(t+1) = 0.$$

2. Acceleration:
   if $((b_{n+1}(t) = 0 \text{ and } b_n(t) = 0) \text{ or } (t_h \geqslant t_s))$ then $v_n(t+1) = \min(v_n(t) + 1, v_{\max})$
   else $v_n(t+1) = v_n(t)$.

[1] As pointed out in [21], the disorder effect in multi-lane traffic has a strong influence on the structure of the traffic states, so it is likely that the light synchronized traffic is reproduced by the CD model if the lane change behaviour and/or the disorder effect are considered.

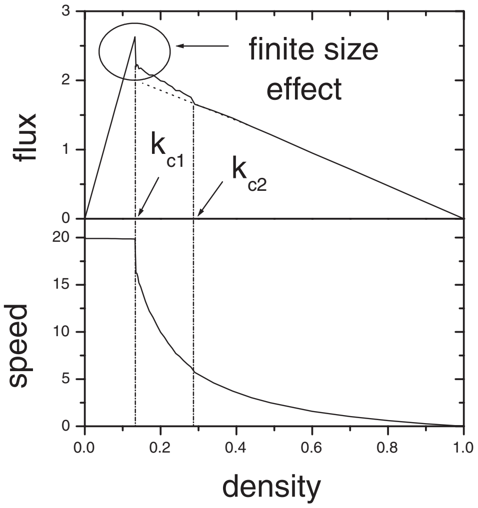

Figure 1. The fundamental diagram of the CD model. The peak of the line is a finite size effect. The dotted line is a guide to the eyes.

3. Braking rule:

$$v_n(t+1) = \min\left(d_n^{\text{eff}}, v_n(t+1)\right)$$
$$\text{if} \quad (v_n(t+1) < v_n(t)) \quad \text{then} \quad b_n(t+1) = 1.$$

4. Randomization and braking:

$$\text{if} \quad (\text{rand}() < p) \quad \text{then} \quad \begin{cases} v_n(t+1) = \max(v_n(t+1) - 1, 0) \\ \text{if} \quad (p = p_b) \quad \text{then} \quad b_n(t+1) = 1. \end{cases}$$

5. Car motion:

$$x_n(t+1) = x_n(t) + v_n(t+1).$$

Here x_n is the position of vehicle n, rand() is a random number between 0 and 1. The model parameters $v_{\max} = 20$, $p_d = 0.1$, $p_b = 0.94$, $p_0 = 0.5$, $h = 6$ and $gap_{\text{safety}} = 7$ are used in [20]. Each cell corresponds to 1.5 m and a vehicle has a length of five cells. One time step corresponds to 1 s.

In figure 1, we show the fundamental diagram of the CD model with the same parameters as in [20]. The simulations are performed on a ring with a length of 10 000 cells. It can be seen that three density ranges can be classified. When the density $k < k_{c1}$, the traffic will be free flow and the speed is the free speed (which is a little smaller than the maximum speed $v_{\max}$ due to the randomization probability p_d). When $k_{c1} < k < k_{c2}$, a phase separation phenomenon will occur and the system is the coexistence of free flow phase and congested flow phase (figure 2(*a*)). Note that the congested flow consists of both synchronized flow and jams. With the increase of the density, the free flow region shrinks (see figures 2(*a*) and (*b*)). When $k > k_{c2}$, the free flow region disappears and only the congested flow exists (figure 2(*c*)). If one continues to increase the density, the jams will gradually invade the synchronized flow (see figures 2(*c*) and (*d*)).

Analogous to the empirical analysis [22], we check the 1 min average data of the CD model. The simulation data are evaluated by a virtual detector. The mean flux J and the mean velocity v of cars passing the detector in a time interval of 1 min are recorded. The density

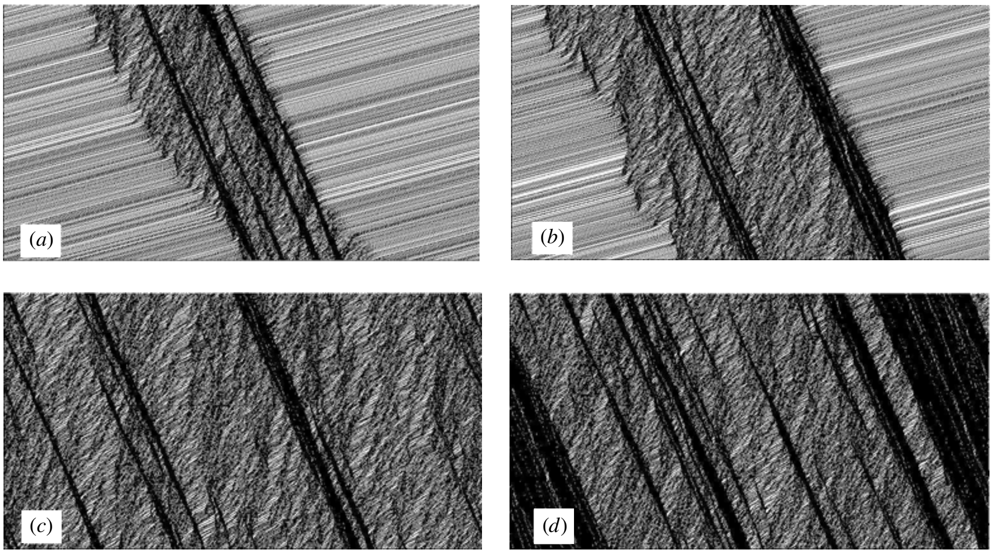

Figure 2. The spacetime plot of the CD model. The cars are moving from left to right, and the vertical direction (up) is (increasing) time. (*a*) $k = 0.2$, (*b*) $k = 0.25$, (*c*) $k = 0.35$ and (*d*) $k = 0.5$.

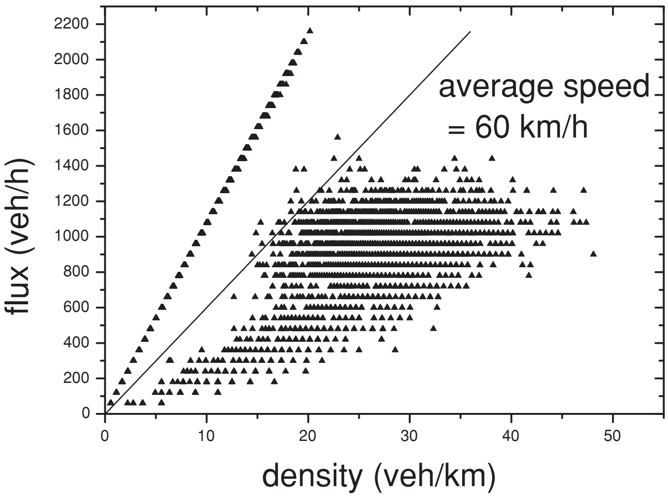

Figure 3. The 1 min average flux against density of the CD model. Note that here the quantities have units. The straight line has a slope of 60 km h^{-1}.

is calculated by $k = J/v$. The result is shown in figure 3, where the data analysis has been performed separately for the free flow and congested flow excluding the transition regime between them as in [22]. One can see that the mean velocity of the synchronized flow that can be described by the CD model is basically not greater than 60 km h^{-1}. This implies that the CD model cannot depict the light synchronized flow that has a quite large average speed [23].

3. The new model

In this section, we present a new CA model that can reproduce not only light and heavy synchronized flows, but also the outflow from the traffic jam.

Before the presentation of the model, we first discuss the concept of the slow-to-start rule. The rule is introduced for the description of the insensitive reaction of the drivers to the preceding car [10]. Generally it is assumed that the drivers of stopped cars are less sensitive than the drivers of moving cars. So the randomization probability of the stopped car is set to be larger than that of the moving car. Nevertheless, we argue that a realistic case should be this: the driver of a car that has just stopped still remains sensitive; only when the car has stopped for a certain time t_c does the driver become less sensitive. Thus, in our new model, the randomization function (1) is varied

$$p(v_n(t), b_{n+1}(t), t_h, t_s) = \begin{cases} p_b: & \text{if} \quad b_{n+1} = 1 \quad \text{and} \quad t_h < t_s \\ p_0: & \text{if} \quad v_n = 0 \quad \text{and} \quad t_{st} \geqslant t_c \\ p_d: & \text{in all other cases} \end{cases} \tag{3}$$

where t_{st} denotes the time that a car stops, and the parameters t_s, t_h are the same as in the CD model.

Now we present our model as follows.

1. Determination of the randomization parameter p:

$$p = p(v_n(t), b_{n+1}(t), t_h, t_s).$$

2. Acceleration:
if $((b_{n+1}(t) = 0 \text{ or } t_h \geqslant t_s) \text{ and } (v_n(t) > 0))$ then $v_n(t+1) = \min(v_n(t) + 2, v_{\max})$
else if $(v_n(t) = 0)$ then $v_n(t+1) = \min(v_n(t) + 1, v_{\max})$
else $v_n(t+1) = v_n(t)$
3. Braking rule:

$$v_n(t+1) = \min\left(d_n^{\text{eff}}, v_n(t+1)\right).$$

4. Randomization and braking:

$$\text{if} \quad (\text{rand}() < p) \quad \text{then} \quad v_n(t+1) = \max(v_n(t+1) - 1, 0).$$

5. The determination of $b_n(t+1)$:

$$\text{if} \quad (v_n(t+1) < v_n(t)) \quad \text{then} \quad b_n(t+1) = 1$$
$$\text{if} \quad (v_n(t+1) > v_n(t)) \quad \text{then} \quad b_n(t+1) = 0$$
$$\text{if} \quad (v_n(t+1) = v_n(t)) \quad \text{then} \quad b_n(t+1) = b_n(t).$$

6. The determination of t_{st}:

$$\text{if} \quad v_n(t+1) = 0 \quad \text{then} \quad t_{st} = t_{st} + 1$$
$$\text{if} \quad v_n(t+1) > 0 \quad \text{then} \quad t_{st} = 0.$$

7. Car motion:

$$x_n(t+1) = x_n(t) + v_n(t+1).$$

In the model, the effective distance d_n^{eff} is the same as in the CD model. In step 2, the acceleration capacity of a stopped car is assumed to be 1 and that of a moving car is 2. We also note that the definition of b_n in the model is different from that in the CD model in two aspects: (i) In this model, the brake light is switched on if the speed decreases. This is different from the CD model, where the brake light has nothing to do with the decrease in speed caused by the randomization probability p_d; (ii) In this model, after it switches on, the brake light will not switch off unless the car begins to accelerate.

The fundamental diagram of the model is shown in figure 4, where t_c is set to 10 and the other parameter values are the same as in section 2. The fundamental diagram has two branches: the upper branch is obtained from the initially homogeneous distribution of traffic whereas the lower branch starts from a megajam. When the density is very low ($k < k_{c4}$), the free

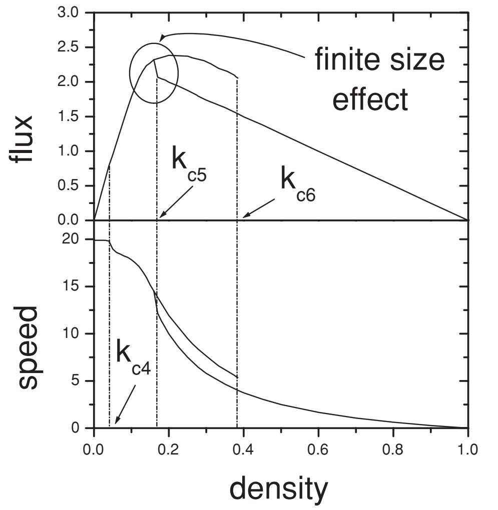

Figure 4. The fundamental diagram of the new model. The peak of the line is a finite size effect.

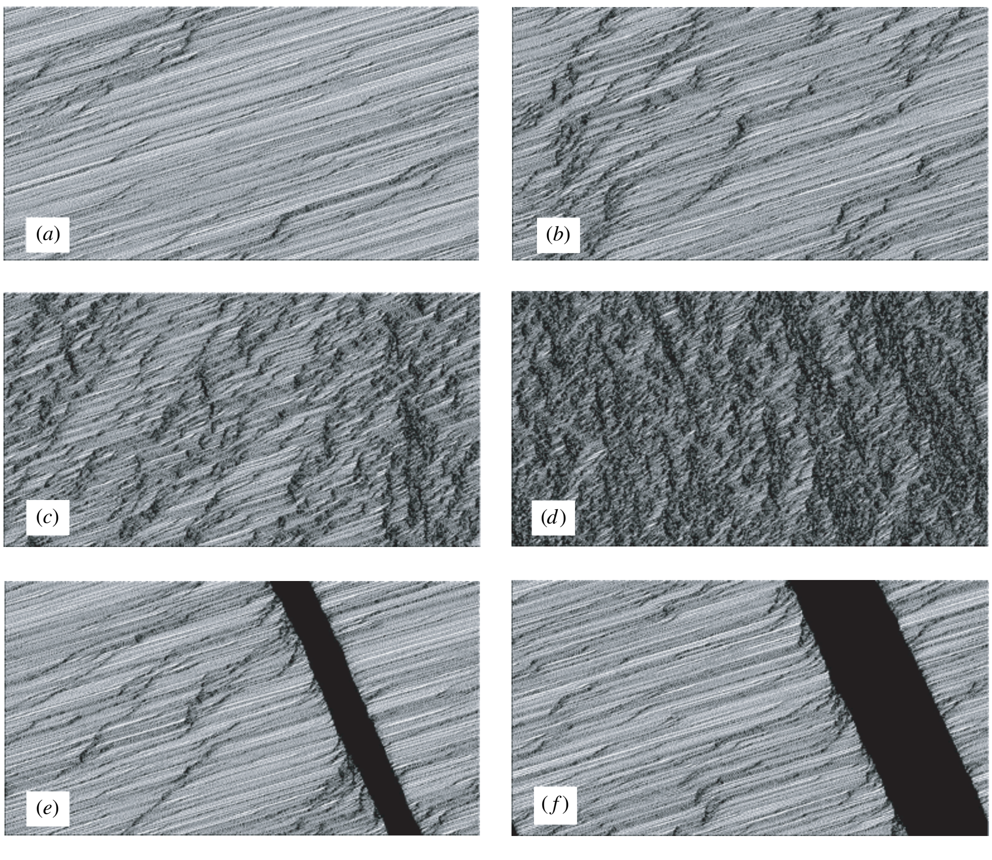

Figure 5. The spacetime plot of the new model. (*a*) $k = 0.12$, (*b*) $k = 0.15$, (*c*) $k = 0.2$, (*d*) $k = 0.3$, (*e*) $k = 0.2$ and (*f*) $k = 0.3$. In (*c*) and (*d*), the traffic starts from a homogeneous distribution, in (*e*) and (*f*), the traffic starts from a megajam.

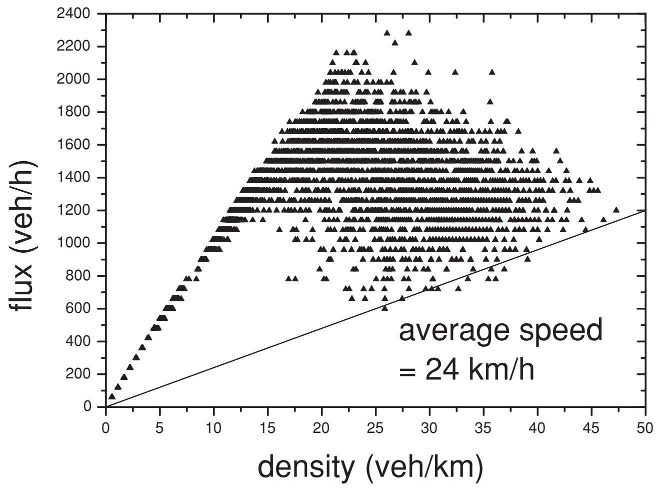

Figure 6. The 1 min average flux against density of the new model. The straight line has a slope of 24 km h^{-1}.

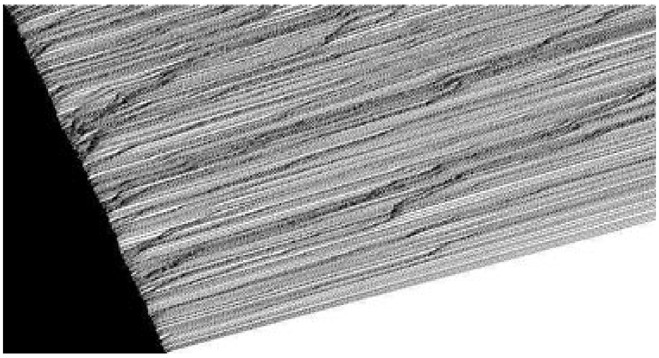

Figure 7. The spacetime plot of the outflow from the jam of the new model.

flow with free speed exists. When the density exceeds k_{c4}, the light synchronized flow begins to emerge (figure 5(*a*)). When the density is in the range $k_{c4} < k < k_{c5}$, the synchronized flow region gradually invades the free flow region and simultaneously, the light synchronized flow gradually transforms into the heavy synchronized flow (figure 5(*b*)). When the density exceeds k_{c5}, the homogeneous traffic will evolve into the synchronized flow (figures 5(*c*) and (*d*)). In contrast, if we start from a megajam, the traffic will be the coexistence of jams, free flow and light synchronized flow (figures 5(*e*) and (*f*)). When the density is large enough, namely, $k > k_{c6}$, any initial distribution will lead to the coexistence of jams, free flow and light synchronized flow.

The 1 min average data of the model is presented in figure 6. One can see that the model can describe light synchronized flow as well as heavy synchronized flow with average speed greater than approximately 24 km h^{-1}. As for heavy synchronized flow with speed lower than 24 km h^{-1}, it is unstable and will evolve into the coexistence of jams, free flow and light synchronized flow. This is consistent with the empirical findings—stop-and-go waves are always self-organized from the synchronized flow [4].

Moreover, the model can simulate the outflow from the jams quite satisfactorily. This can be seen from figure 7.

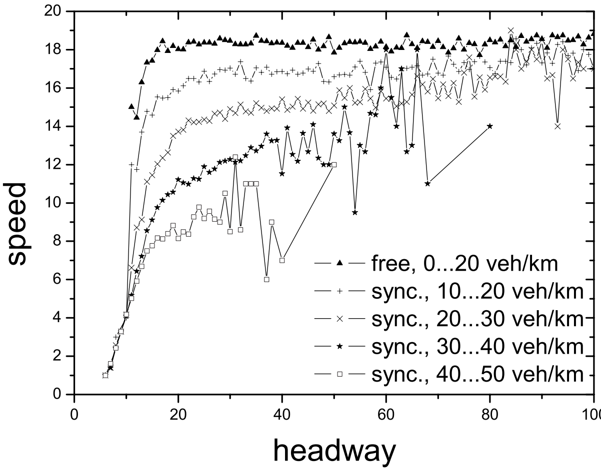

Figure 8. The dependence of the mean speed chosen by a driver on the global traffic state and the gap between the preceding car.

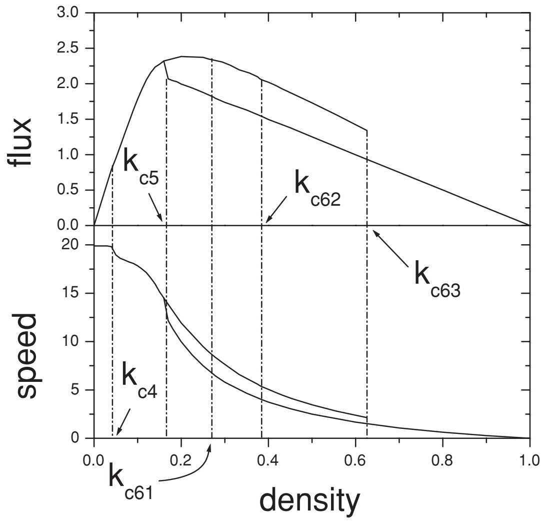

Figure 9. The fundamental diagram of the new model under different t_c. k_{c61}, k_{c62} and k_{c63} denote the critical density k_{c6} for $t_c = 8$, 10 and 12, respectively.

We consider the velocity–headway curve, which illustrates the vehicle's velocity adjustment on the distance headway. In the free flow, cars can move freely, thus the velocity saturates even for small distances. With increasing density, the vehicle interaction strengthens and cars tend to have lower velocities than the headway allows. As a result, the asymptotic value of the velocity decreases (see figure 8). This is consistent with the empirical observations [22, 24].

Finally, we investigate the effect of the time t_c on the traffic flow. The simulations show that t_c has no influence on the lower branch of the fundamental and the upper branch in the density range $k < k_{c5}$. In contrast, the critical density k_{c6} depends on t_c. A larger t_c leads to a higher k_{c6} (see figure 9). When $t_c = 1, k_{c6}$ becomes identical with k_{c5}. This implies that the longer the drivers can remain sensitive, the heavier the synchronized flow can be maintained.

4. Conclusions

Due to the simplicity and the easy implementation on computers for numerical investigations, the CA traffic flow models developed very quickly in the last decade. However, to our knowledge, despite its success in modelling traffic flow, the CA models cannot reproduce synchronized flow.

In this paper, our aim is to deliver a better understanding of the simulation of synchronized flow using the CA models. To do so, we present a new CA model. The model can describe the outflow from the jams, the light synchronized flow as well as heavy synchronized flow with average speed greater than approximately 24 km h^{-1}. As for the synchronized flow with speed lower than 24 km h^{-1}, it is unstable and will evolve into the coexistence of jams, free flow and light synchronized flow. This is consistent with the empirical findings.

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